Math 365 – Quiz 3 – Monday 3/25/19

Solutions
Name

Instructions: You may use the choose functions $\binom{n}{k}$ and Sterling functions $S(n,k)$ in your answers, and you should leave your answers unsimplified. For example, an answer of the form $\binom{5}{3} * 7 - 5 * S(3,2)$ is acceptable. Recall that a standard deck of cards has 4 suits (clubs, diamonds, hearts, and spades), with 13 cards per suit, having 52 cards in total.

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1. How many ways are there to choose a hand of 5 cards from a standard deck, resulting in five cards of the same suit?

pick suit: 4 * pick 5 cards from that suit: $\binom{13}{5}$

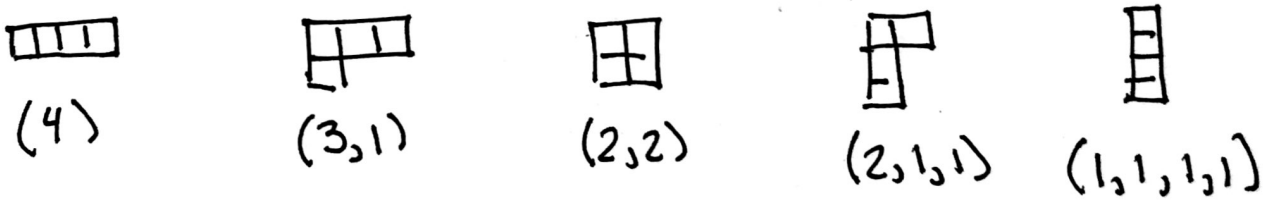
$$\boxed{4\binom{13}{5}}$$

2. How many non-negative integer solutions are there to the equation $x_1 + x_2 + x_3 = 25$?

25 indist obj into 3 dist boxes

$$\boxed{\binom{25+3-1}{25}}$$

3. List the integer partitions of 4.



(4) (3,1) (2,2) (2,1,1) (1,1,1,1)

4. How many ways are there to fill 7 identical offices with 10 people (so that all the offices are occupied)?

10 dist obj into 7 indist boxes

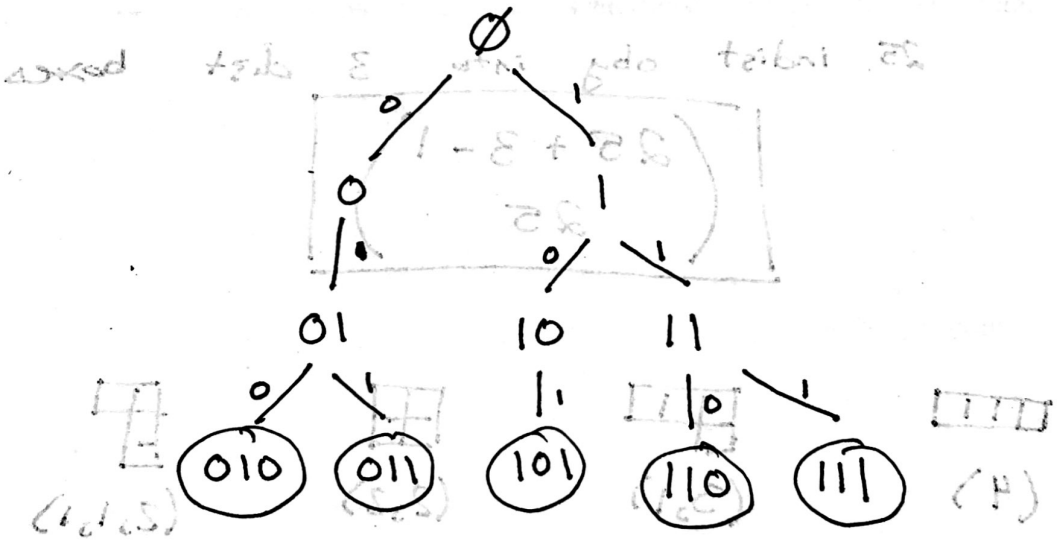
$$S(10,7)$$

5. State the binomial theorem.

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

6. Draw a decision tree for the following counting problem, and use it to give the answer in the box provided:

How many strings of length-three of 1's and 0's do not have two consecutive 0's?



Answer:

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