SOLUTIONS
Homework 2
Due 2/13/19

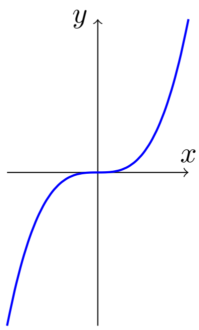
Note for those of you learning to write in LaTeX:
I have published both the .pdf and .tex files for these solutions on the course website. Just like with the .tex files for the exercised each day, please feel free to use my code to help you learn to write in LaTeX! You should never copy answers from anywhere for your assignments, but you're welcome to copy code that helps you to format and communicate your own ideas as you write.

Exercise 8. (a) Decide for each of the following expressions: Is it a function? If so,

(i) what is its domain, codomain, and image? (ii) is it injective? (why or why not)
(iii) is it surjective? (why or why not) (iv) is it invertible? (why or why not)

(I) $f : \mathbb{R} \to \mathbb{R}$ defined by $x \mapsto x^3$

Answer. Yes, f is a function. Recall the graph of $y = x^3$.



(i) Domain: $\mathbb{R}$. Codomain: $\mathbb{R}$. Range: $\mathbb{R}$.
(ii) Yes injective: for all $x_1, x_2 \in \mathbb{R}$, if $x_1^3 = x_2^3$, then $x_1 = x_2$.
(iii) Yes surjective: for all $y \in \mathbb{R}$ there is some $x \in \mathbb{R}$ such that $y = x^3$.
(iv) Yes invertible: f is bijective, and therefore invertible.

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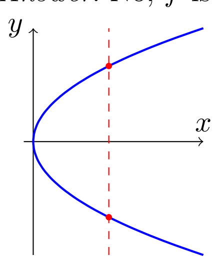
(II) $f : \mathbb{R} \to \mathbb{R}$ defined by $x \mapsto \sqrt{x}$

Answer. No, f is not a function:



This is because $g(x) = x^2$ is not an injective function; i.e. since $(-2)^2 = 4$ and $2^2 = 4$, we have $\sqrt{4} = \pm 2$, which is not a single value.

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(III) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Q}$ defined by $(a, b) \mapsto a/b$

Answer. No, f is not a function. In particular, for any $a \in \mathbb{Z}$, $f((a, 0))$ is not defined.

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(IV) $f : \mathbb{R} \times \mathbb{Z} \to \mathbb{Z}$ defined by $(r, z) \mapsto \lceil r \rceil * z$

Answer. Yes, f is a function.

(i) Domain: $\mathbb{R} \times \mathbb{Z}$. Codomain: $\mathbb{Z}$.
Range: $\mathbb{Z}$, since $f((1, z)) = z$ for all $z \in \mathbb{Z}$.
(ii) Not injective: for example, $f((1, 1)) = 1 = f((-1, -1))$.
(iii) Yes surjective: the codomain is the same as range.
(iv) Not invertible: f is not injective, and therefore is not invertible.

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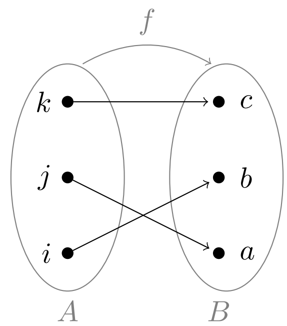
(V)



Answer. Yes, f is a function.

(i) Domain: $\{i, j, k\}$. Codomain: $\{a, b, c\}$. Range: $\{a, b, c\}$

(ii) Yes injective: for all $x \in \{i, j, k\}$, $f^{-1}(x)$ has exactly one element, so f is a bijection. Thus f is injective.

(iii) Yes surjective: see (ii).

(iv) Yes invertible: f is a bijection.

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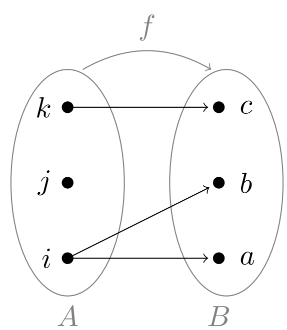
(VI)



Answer. No, f is not a function, both since j doesn't have an image and i has more than one image.

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(b) For those examples which were not functions in the previous problems, can you restrict the domain and/or codomain to make them functions? i.e. pick a reasonable subset of the given domain/codomain such that the expression is a function on that domain.

Answer.

(II) Some examples: Restrict the domain to $\mathbb{R}_{\geq 0}$, or to $\mathbb{R}_{\leq 0}$, or to $\{1\}$, or. . . .

(III) Some examples: Restrict the domain to $\mathbb{R} \times \mathbb{Z}_{>0}$, or to $\mathbb{R} \times (\mathbb{Z}_{>0} \cup \mathbb{Z}_{<0}$, or to $\{-3\} \times \{2\}$, or. . . .

(VI) Restrict the domain to $\{k\}$ (this is the only possibility).

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(c) If possible, choose a domain and codomain for the following expressions to make them into functions satisfying

(i) f is an injective but not surjective function;
(ii) f is a surjective but not injective function;
(iii) f is an invertible function;
(iv) f is not a function;

where f is given by:

(I) $f(x) = |x|$

Answer.

(i) Injective but not surjective: Domain $\mathbb{R}_{>0}$, Codomain $\mathbb{R}$.
(ii) Surjective but not injective: Domain $\mathbb{R}$, Codomain $\mathbb{R}_{\geq 0}$.
(iii) Invertible function: Domain $\mathbb{R}_{>0}$, Codomain $\mathbb{R}_{>0}$.
(iv) Not a function: Domain $\mathbb{R}_{>0}$, Codomain $\{0\}$.

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(II) $f(x) = 1/x$

Answer.

(i) Injective but not surjective: Domain $\mathbb{R}_{>0}$, Codomain $\mathbb{R}$.
(ii) Surjective but not injective: Not possible within $\mathbb{C}$.
(iii) Invertible function: Domain $\mathbb{R}_{>0}$, Codomain $\mathbb{R}_{>0}$.
(iv) Not a function: Domain $\mathbb{R}$, Codomain anything.

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(III) $f(x) = 6$

Answer.

(i) Injective but not surjective: Domain $\{1\}$, Codomain $\mathbb{R}$.
(ii) Surjective but not injective: Domain $\{1, 2\}$, Codomain $\{6\}$.
(iii) Invertible function: Domain $\{1\}$, Codomain $\{6\}$.
(iv) Not a function: Domain $\{1, 2\}$, Codomain $\{0\}$.

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(IV) $f(x, y) = \sqrt{xy}$.

Answer.

(i) Injective but not surjective: Domain $\mathbb{Z}_{>0} \times \{1\}$, Codomain $\mathbb{R}$.
(ii) Surjective but not injective: Domain $\mathbb{Z}_{>0} \times \{0\}$, Codomain $\{0\}$.
(iii) Invertible function: Domain $\{0\} \times \{0\}$, Codomain $\{0\}$.
(iv) Not a function: Domain $\{1\} \times \{1\}$, Codomain $\{\pm 1\}$.

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(d) Let f be a function from the set A to the set B. Let S and T be subsets of A. Show that

(i) $f(S \cup T) = f(S) \cup f(T)$:

Proof. First, if $x \in f(S \cup T)$, then $x = f(a)$ for $a \in S \cup T$. So either $x = f(a)$ for $a \in S$, or $x = f(a)$ for $a \in T$. Thus $x \in f(S) \cup f(T)$, implying that $f(S \cup T) \subseteq f(S) \cup f(T)$.

Conversely, if $x \in f(S) \cup f(T)$, then $x \in f(S) \subseteq f(S \cup T)$ or $x \in f(T) \subseteq f(S \cup T)$. So $x \in f(S \cup T)$. Thus $f(S \cup T) \supseteq f(S) \cup f(T)$.

Therefore $f(S \cup T) = f(S) \cup f(T)$. ☐

(ii) $f(S \cap T) \subseteq f(S) \cap f(T)$.

Proof. If $x \in f(S \cap T)$ then $x = f(a)$ for $a \in S$ and $a \in T$. So $x \in f(S)$ and $x \in f(T)$, implying $x \in f(S) \cap f(T)$. Thus $f(S \cap T) \subseteq f(S) \cap f(T)$. ☐

Can you think of an example where the inclusion in part (b) could be proper (not equal)?

Answer. For example, let

$$f : \{a, b\} \to \{x\} \quad \text{be the function} \quad f(a) = f(b) = x.$$

With $S = \{a\}$ and $T = \{b\}$,

$$f(S \cap T) = f(\emptyset) = \emptyset$$

and

$$f(S) \cap f(T) = \{x\} \cap \{x\} = \{x\} \supsetneq \emptyset.$$

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Exercise 9. Let

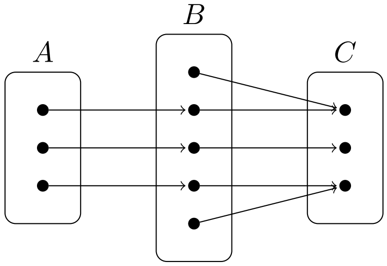
$$f : A \to B \qquad \text{and} \qquad g : B \to C$$

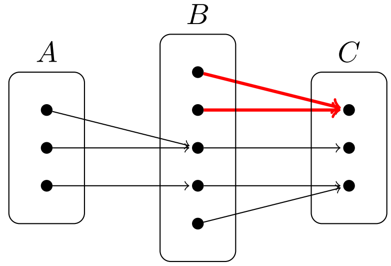
be functions. Draw some pictures and make some conjectures about the following questions.

(a) Is $g \circ f$ always a function? *Answer:* Yes.

(b) What are the conditions on f, g, A, B, and/or C for $g \circ f$ to be surjective?

Answer. It must be the case that (1) g is surjective, and (2) for each $c \in C$, $f(A) \cap g^{-1}(c)$ must be nonempty.

Good: 

Bad: 

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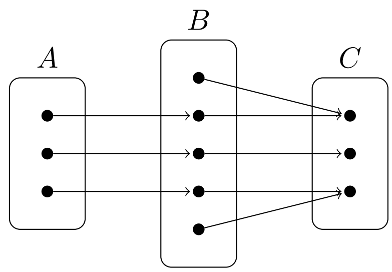
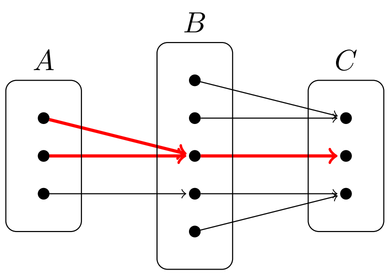
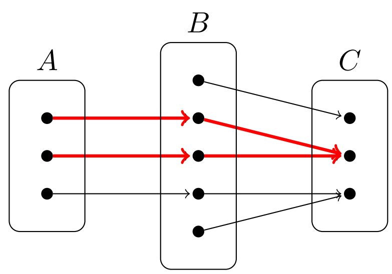
(c) What are the conditions on f, g, A, B, and/or C for $g \circ f$ to be injective?

Answer. It must be the case that (1) f is injective, and (2) that g restricted to $f(A)$ must also be injective.

Good: 

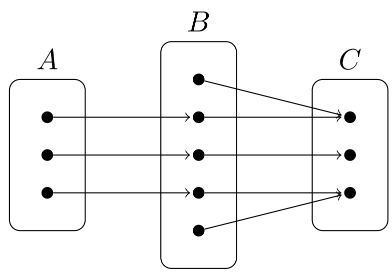
Bad: 

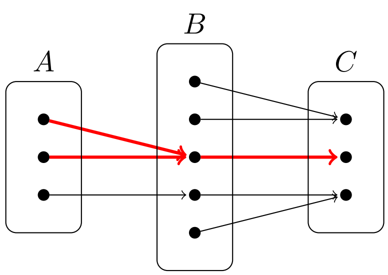
Bad: 

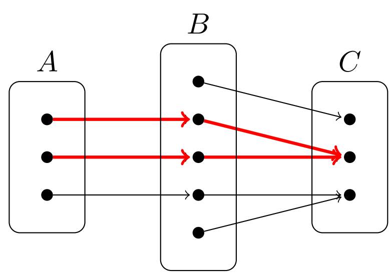
. . . .

(d) What are the conditions on f, g, A, B, and/or C for $g \circ f$ to be bijective?

Answer. In this case, (1) f must be injective, and (2) g restricted to $f(A)$ must be bijective.

Good: 

Bad: 

Bad: 

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(e) What are the conditions on f, g, A, B, and/or C for $f \circ g$ to be a function?

Answer. In order for f to be defined on $g(B)$, we must have $g(B) \subseteq A$.

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(f) If f and $g \circ f$ are injective, is it necessarily true that g is injective?

Answer. No. For example, if

$$\begin{aligned} f : \{1\} &\to \{2,3\} \\ 1 &\mapsto 2 \end{aligned} \qquad \text{and} \qquad \begin{aligned} g : \{2,3\} &\to \{4\} \\ 2 &\mapsto 4 \\ 3 &\mapsto 4 \end{aligned}$$

then

$$\begin{aligned} g \circ f : \{1\} &\to \{4\} \\ 1 &\mapsto 4 \end{aligned}$$

is injective even though g isn't.

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(g) If f and $g \circ f$ are surjective, is it necessarily true that g is surjective? *Answer:* Yes: see (b).

Exercise 10. Show that if both

$$f : A \to B \qquad \text{and} \qquad g : B \to C$$

are surjective functions, then $g \circ f$ is also surjective.

Proof. Pick any $c \in C$. Since g is a surjective function, there is some $b \in B$ such that $g(b) = c$. And since f is a surjective function, there is some $a \in A$ such that $f(a) = b$. So $g(f(a)) = g(b) = c$, so that $c \in (g \circ f)(A)$. Since this was true for *any* $c \in C$, we have $C \subseteq (g \circ f)(A)$. Therefore $g \circ f$ is surjective. $\square$

Exercise 11.

(a) Determine whether f is a function from $\mathbb{Z}$ to $\mathbb{R}$ if

(i) $f(n) = \pm n$: *Answer:* No, since $1 \mapsto 1$ and $1 \mapsto -1$.

(ii) $f(n) = \sqrt{n^2 + 1}$: *Answer:* Yes, if we always take the positive root, no if not.

(iii) $f(n) = 1/(n^2 - 4)$: *Answer:* No, since $f(2)$ is not defined.

(b) Let $f(x) = 2x$ where the domain is the set of real numbers. Compute the following images.

(i) $f(\mathbb{Z})$ *Answer:* { even integers }

(ii) $f(\mathbb{N})$ *Answer:* { positive even integers }

(iii) $f(\mathbb{R})$ *Answer:* $\mathbb{R}$

(c) Let f be the function from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = x^2$. Use set-builder notation to describe the following preimages.

(i) $f^{-1}(\{1\})$ *Answer:* $\{-1, 1\}$

(ii) $f^{-1}(\{x \mid 0 < x < 1\})$ *Answer:* $\{x \in \mathbb{R} \mid -1 < x < 1 \text{ and } x \neq 0\}$

(iii) $f^{-1}(\{x \mid x > 7\})$ *Answer:* $\{x \in \mathbb{R} \mid x < -|\sqrt{7}| \text{ or } x > |\sqrt{7}|\}$

Exercise 12.

(a) For each of the following sequences, compute the terms a_0, a_1, a_2, and a_3.

(a) $a_n = 3$;

Answer. We have

$$a_0 = 3, \quad a_1 = 3, \quad a_2 = 3, \quad \text{and} \quad a_3 = 3.$$

....

(b) $a_n = 7 + 4^n$;

Answer. We have

$$a_0 = 8, \quad a_1 = 11, \quad a_2 = 23, \quad \text{and} \quad a_3 = 71.$$

....

(c) $a_n = 2^n + (-2)^n$.

Answer. We have

$$a_0 = 2, \quad a_1 = 0, \quad a_2 = 8, \quad \text{and} \quad a_3 = 0.$$

....

(b) For each of the following sequences defined by recurrence relations and initial conditions, answer the following.

(a) Compute the first four terms (a_0, a_1, a_2, and a_3).

(b) Decide if $\{a_n\}$ is arithmetic, geometric, or neither. If it is arithmetic or geometric, then find a closed formula for a_n.

(i) The sequence satisfying $a_0 = 2$ and $a_n = \frac{1}{2}a_{n-1}$.

Answer. We have

$$a_0 = 2, \quad a_1 = 1, \quad a_2 = 1/2, \quad \text{and} \quad a_3 = 1/4.$$

Since $a_n - a_{n-1}$ is not constant, this sequence is not arithmetic. But since $a_n/a_{n-1} = 1/2$ is constant, this sequence is geometric.
The closed formula is

$$a_n = a_0 * r^n = 2(1/2)^n.$$

....

(ii) The sequence satisfying $a_0 = -1$ and $a_n = a_{n-1} + 5$.

Answer. We have

$$a_0 = -1, \quad a_1 = 4, \quad a_2 = 9, \quad \text{and} \quad a_3 = 14.$$

Since $a_n - a_{n-1} = 5$ is constant, this sequence is arithmetic. But since a_n/a_{n-1} is not constant, this sequence is not geometric.
The closed formula is

$$a_n = a_0 + cn = -1 + 5n.$$

....

(iii) The sequence satisfying $a_0 = 1$, $a_1 = -1$ and $a_n = a_{n-2} * a_{n-1}$.

Answer. We have

$$a_0 = 1, \quad a_1 = -1, \quad a_2 = -1, \quad \text{and} \quad a_3 = 1.$$

Since $a_1 - a_0 = -2 \neq a_2 - a_1$, the difference of consecutive terms is not constant, so this sequence is not arithmetic. And since $a_1/a_0 = -1 \neq a_2/a_1$, the ratio of consecutive terms is not constant, so this sequence is not geometric.

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(iv) The sequence satisfying $a_0 = 2$ and $a_n = -a_{n-1}$.

Answer. We have

$$a_0 = 2, \quad a_1 = -2, \quad a_2 = 2, \quad \text{and} \quad a_3 = -2.$$

Since $a_n - a_{n-1}$ is not constant, this sequence is not arithmetic. But since $a_n/a_{n-1} = -1$ is constant, this sequence is geometric.
The closed formula is

$$a_n = a_0 * r^n = 2(-1)^n.$$

. . . .

(c) Find at least three different sequences beginning with the terms $3, 5, 7$ whose terms are generated by a simple formula or rule.

Answer. Some examples, each beginning with $n = 0$:

$a_n = 2n + 3;$
$a_n = a_{n-1} + 2\left\lceil \frac{n}{2} \right\rceil, \; a_0 = 3;$
$a_0 = 3, \; a_1 = 5, \; a_2 = 7, \;$ and $a_n = 1$ for $n \geq 3;$
$a_n = a_{n-1} + 2f_n, \; a_0 = 3,$ where f_n is the Fibbonacci sequence starting with $f_0 = 0, f_1 = 1;$
$a_n =$ the day value of dates that MWFs fall on, beginning with August 3rd, 2015.

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(d) For the following sequences, try to find the pattern. Decide if they are arithmetic, geometric, or neither. If it's arithmetic or geometric, find a closed formula expressing the nth term of the sequence.

(i) $5, 1, -3, -7, -11, \ldots$

Answer. We have

$$-4 = 1 - 5 = -3 - 1 = -7 - (-3) = -11 - (-7),$$

so this appears to be an arithmetic sequence with difference 4. If so,

$$a_n = 5 - 4n, \qquad \text{for } n = 0, 1, 2 \ldots.$$

. . . .

(ii) $1, 4, 9, 16, 25, \ldots$

Answer. We have

$$4 - 1 \neq 9 - 4 \quad \text{and} \quad 4/1 \neq 9/4,$$

so this sequence is neither arithmetic nor geometric. But it does appear to follow the pattern

$$a_n = n^2, \qquad \text{for } n = 1, 2, 3 \ldots.$$

. . . .

(iii) $3, 9, 27, 81, 243, \ldots$

Answer. We have

$$3 = 9/3 = 27/9 = 81/27, 243/81$$

so this appears to be a geometric sequence with ratio 3. If so,

$$a_n = 3^n, \qquad \text{for } n = 1, 2, 3 \ldots.$$

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(e) Show that both of the following sequences are solutions to the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ with initial condition $a_0 = 1$.

(i) $a_n = 1$:

Proof. We have $a_0 = 1$ and

$$-3a_{n-1} + 4a_{n-2} = -3 * 1 + 4 * 1 = 1 = a_n,$$

as desired.

□

(ii) $a_n = (-4)^n$:

Proof. We have $a_0 = (-4)^0 = 1$ and

$$\begin{aligned} -3a_{n-1} + 4a_{n-2} &= -3(-4)^{n-1} + 4(-4)^{n-2} \\ &= (-4)^{n-2}\,(-3(-4) + 4) \\ &= (-4)^{n-2} * 16 \\ &= (-4)^{n-2}(-4)^2 \\ &= (-4)^n, \end{aligned}$$

as desired.

□

Exercise 13. (a) Compute the first three partial sums for the following infinite series.

(i) $\displaystyle\sum_{i=0}^{\infty} 5i+1$:

Answer.

$$\sum_{i=0}^{0} 5i+1 = 5*0+1 = 1;$$
$$\sum_{i=0}^{1} 5i+1 = (5*0+1)+(5*1+1) = 1+6 = 7;$$
$$\sum_{i=0}^{2} 5i+1 = (5*0+1)+(5*1+1)+(5*2+1) = 1+6+11 = 18.$$

. . . .

(ii) $\displaystyle\sum_{i=4}^{\infty} i(i+1)$:

Answer.

$$\sum_{i=4}^{4} i(i+1) = 4*5 = 20;$$
$$\sum_{i=4}^{5} i(i+1) = 4*5+5*6;$$
$$\sum_{i=4}^{6} i(i+1) = 4*5+5*6+6*7.$$

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(b) Calculate the following.

(i) $\displaystyle\sum_{j=0}^{8} (1+(-1)^j)$

Answer.

$$\sum_{j=0}^{8} (1+(-1)^j) = 2+0+2+0+2+0+2+0+2 = \boxed{10}$$

. . . .

(ii) $\displaystyle\sum_{j=-1}^{2}\sum_{i=2}^{3}(2i+3j)$

Answer.

$$\begin{aligned}
\sum_{j=-1}^{2}\sum_{i=2}^{3}(2i+3j) &= \sum_{j=-1}^{2}((2*2+3j)+(2*3+3j)) \\
&= \sum_{j=-1}^{2}(10+6j) \\
&= (10+6(-1))+(10+6(0))+(10+6(1))+(10+6(2)) \\
&= 40+6(-1+0+1+2) = \boxed{52}
\end{aligned}$$

. . . .

(iii) $\displaystyle\sum_{j\in\{-1,4,15\}} 2$

Answer.

$$\sum_{j\in\{-1,4,15\}} 2 = 2+2+2 = \boxed{6}$$

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(iv) $\displaystyle\sum_{j\in\left\{z\in\mathbb{Z}\;\middle|\;|z|\le 2\right\}} j$

Answer. Since $\{z\in\mathbb{Z} \mid |z|\le 2\} = \{-2,-1,0,1,2\}$, we have

$$\sum_{j\in\{z\in\mathbb{Z}\,\mid\,|z|\le 2\}} j = -2+-1+0+1+2 = \boxed{0}.$$

. . . .

(v) $\displaystyle\sum_{n=2}^{5} a_n - a_{n-1}$ where $a_n = n!$

Answer.

$$\sum_{n=2}^{5}(a_n-a_{n-1}) = (a_2-a_1)+(a_3-a_2)+(a_4-a_3)+(a_5-a_4) = a_5-a_1 = 5!-1! = \boxed{5!-1}$$

. . . .

(vi) $\displaystyle\sum_{i=1}^{2000} i$ *Answer:* $\sum_{i=1}^{2000} i = 2000*2001/2 = \boxed{2,001,000}$

(vii) $\displaystyle\sum_{i=0}^{10}\frac{2}{3^i}$ *Answer:* $\displaystyle\sum_{i=0}^{10}\frac{2}{3^i} = \boxed{2\left(\frac{(1/3)^{11}-1}{1/3-1}\right)}$

(viii) $\displaystyle\sum_{i=0}^{\infty} \frac{2}{3^i}$

Answer.

$$\sum_{i=0}^{\infty} \frac{2}{3^i} = \lim_{n\to\infty} \sum_{i=0}^{n} \frac{2}{3^i} = \lim_{n\to\infty} 2\left(\frac{(1/3)^n - 1}{1/3 - 1}\right) = 2(1/(2/3)) = \boxed{3}.$$

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Exercise 14. A little more.

(a) For each of these sequences find a recurrence relation satisfied by this sequence. (The answers are not unique because there are infinitely many different recurrence relations satisfied by any sequence.)

(i) $a_n = 2n + 3$: *Answer:* $a_n = a_{n-1} + 2$

(ii) $a_n = 5^n$: *Answer:* $a_n = 5a_{n-1}$

(iii) $a_n = n^2$:

Answer. As an example, we know

$$a_n - a_{n-1} = n^2 - (n-1)^2 = n^2 - (n^2 - 2n + 1) = 2n - 1.$$

So this sequence satisfies the recursion relation $\boxed{a_n = a_{n-1} + 2n - 1}$.

. . . .

(b) Use partial sums to explain why, for any sequence $a_0, a_1, \ldots, a_n$, that

$$\sum_{i=1}^{n} a_i - a_{i-1} = a_n - a_0.$$

[Let $S_n = \sum_{i=1}^{n} a_i - a_{i-1}$. Write out S_1, S_2, S_3, and so on until you see the pattern. Then use the fact that $S_n = S_{n-1} + (a_n - a_{n-1})$.]

Answer. Let $S_n = \sum_{i=1}^{n} a_i - a_{i-1}$, so that

$$\begin{aligned} S_1 &= a_1 - a_0 \\ S_2 &= a_1 - a_0 + a_2 - a_1 = a_2 - a_0 \\ S_3 &= a_1 - a_0 + a_2 - a_1 + a_3 - a_2 = a_3 - a_0 \\ &\vdots \end{aligned}$$

Note that $S_n = S_{n-1} + (a_n - a_{n-1})$. So starting with $S_1 = a_1$, adding the next term to S_{n-1} to get S_n always cancels the highest index term of S_{n-1} (i.e. a_{n-1}) and adds a_n. What's left is $a_n - a_0$.

. . . .

(c) Read in section 2.4 about product notation $\prod_{i=m}^{n} a_i$. Then, what are the values of the following products?

(a) $\displaystyle\prod_{i=0}^{10} i$: *Answer:* $0 * 1 * 2 * \cdots * 10 = \boxed{0}$.

(b) $\displaystyle\prod_{i=5}^{8} i(i+1)$: *Answer:* $(5 * 6) * (6 * 7) * (7 * 8) * (8 * 9) = \boxed{5,080,320}$.

(c) $\displaystyle\prod_{i=1}^{100}(-1)^i$.

Answer. Using exponentiation rules, this is

$$\prod_{i=1}^{100}(-1)^i = (-1)^{\sum_{i=1}^{100} i} = (-1)^{100*101/2} = (-1)^{50*101} = \boxed{1}.$$

. . . .

(d) Express $n!$ using product notation. *Answer:* $n! = \prod_{i=1}^{n} i$.

Attach at the end of Homework 2:

At the end of your write-up, include the following, labeling this as **"Writing exercise"**.

(a) Mark up your finished homework assignment, showing where you followed or failed to follow the mechanical and stylistic issues outlined in the handout *Communicating Mathematics through Homework and Exams.* This means **treat your write-up as a second-to-last draft**, and go point-by-point through the handout and address instances where you followed or did not follow each direction in your writing. Use a different-colored pen if you have one, and hand in this marked up draft. You do not need to rewrite the result.

How did you improve this week over homework 1? How might you improve in the future?

(b) List three or more ways that you succeeded or failed at following the advice in *Some Guidelines for Good Mathematical Writing.* How did you improve this week over homework 1? How might you improve in the future?

To receive credit for this assignment, you must complete this exercise. **To receive any credit for homework 2, you must do this writing exercise.**