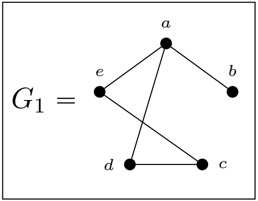
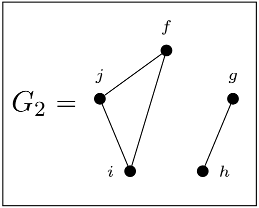
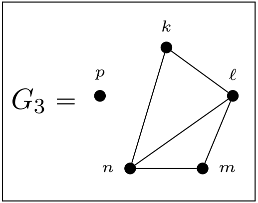
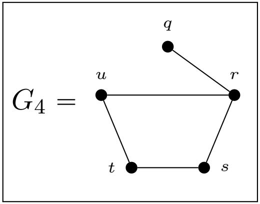
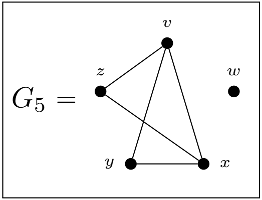
Warmup: Find which pairs of the following graphs are isomorphic. For any two graphs that are isomorphic, give an isomorphism.

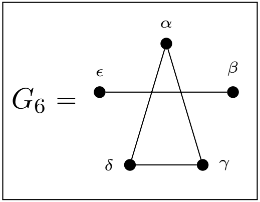












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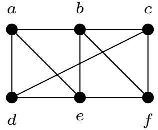
$G =$ 

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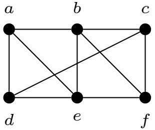

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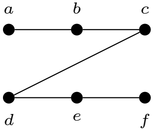

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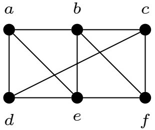

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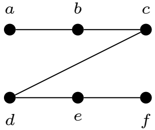
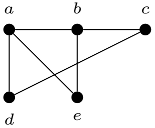


$H_2 =$



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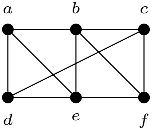
$G =$ 

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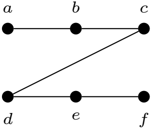
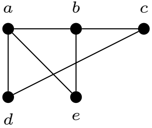
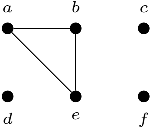
$H_1 =$ 

$H_2 =$ 

$H_3 =$ 

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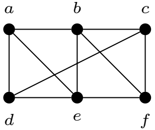
$G =$ 

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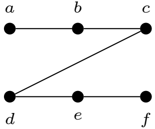
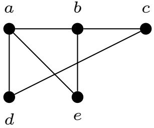
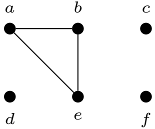
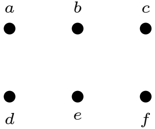
$H_1 =$ 

$H_2 =$ 

$H_3 =$ 

$H_4 =$ 

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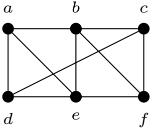

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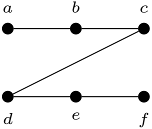

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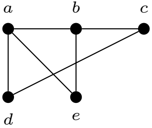

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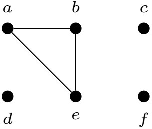

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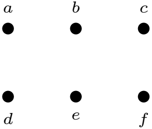

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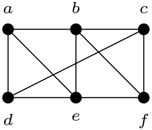
$G =$ 

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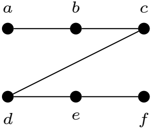
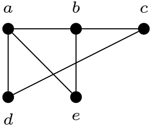
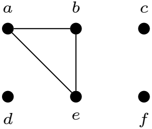
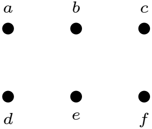
$H_1 =$ 

$H_2 =$ 

$H_3 =$ 

$H_4 =$ 

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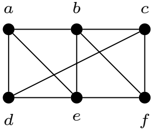
$G =$ 

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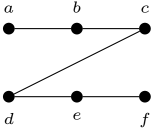
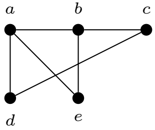
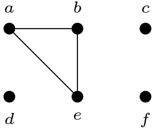
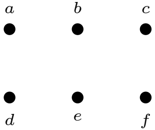
$H_1 =$ 

$H_2 =$ 

$H_3 =$ 

$H_4 =$ 

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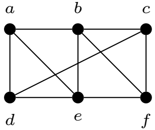
$H_7 =$ 

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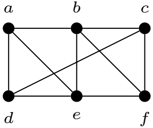
$G =$ 

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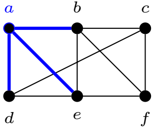
$G =$ 

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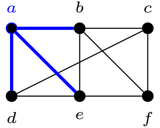
$G =$ 

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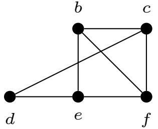
$G - a =$ 

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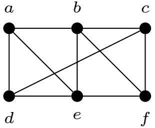
$G =$ 

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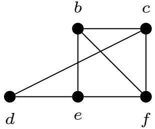
$G - a =$ 

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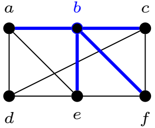
$G =$ 

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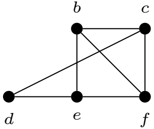
$G - a =$ 

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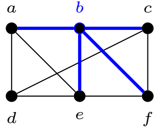
$G =$ 

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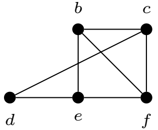
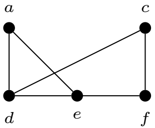
$G - a =$ 

$G - b =$ 

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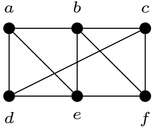
$G =$ 

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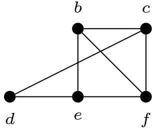
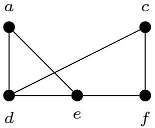
$G - a =$ 

$G - b =$ 

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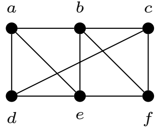
$G =$ 

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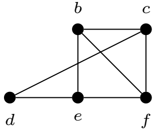
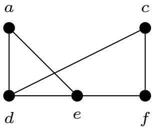
$G - a =$ 

$G - b =$ 

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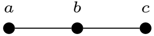
For example, let $G =$ 

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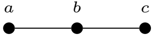
For example, let $G =$ 

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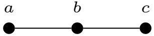
For example, let $G =$ 

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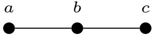
For example, let $G =$ 

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If $H \subseteq G$, then $V_H \subseteq \{a, b, c\}$. Possibilities:

$$\emptyset, \quad \{a\}, \quad \{b\}, \quad \{c\}, \quad \{a, b\},$$
$$\{a, c\}, \quad \{b, c\}, \quad \text{or} \quad \{a, b, c\}.$$

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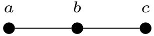
For example, let $G =$ 

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$$\emptyset, \quad \{a\}, \quad \{b\}, \quad \{c\}, \quad \{a, b\},$$
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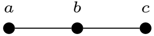
Step 2: Draw the induced graphs for each vertex subset.

Counting subgraphs

Step 1: Break into cases based on the vertex set.

For example, let $G =$ 

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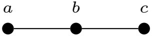
For example, let $G =$ 

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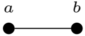
$G[\{a, b\}] =$ 

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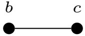
, $G[\{b, c\}] =$ 

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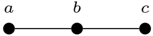
For example, let $G =$ 

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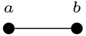
$G[\{a, b\}] =$ 

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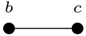
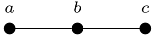
, $G[\{b, c\}] =$ 

$G[\{a, b, c\}] = G =$ 

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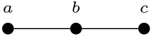
For example, let $G =$ 

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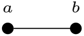
$G[\{a,b\}] =$ 

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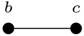
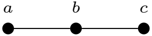
, $G[\{b,c\}] =$ 

$G[\{a,b,c\}] = G =$ 

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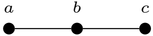
For example, let $G =$ 

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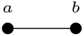
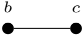
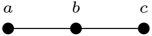
$G[\{a, b\}] =$ , $G[\{a, c\}] =$ , $G[\{b, c\}] =$ 

$G[\{a, b, c\}] = G =$ .

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Counting subgraphs

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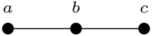
For example, let $G =$ 

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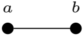
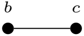
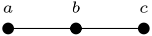
$G[\{a,b\}] =$ , $G[\{a,c\}] =$ , $G[\{b,c\}] =$ 

$G[\{a,b,c\}] = G =$ .

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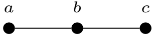
For example, let $G =$ 

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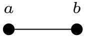
$G[\{a,b\}] =$ 

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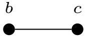
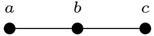
, $G[\{b,c\}] =$ 

$G[\{a,b,c\}] = G =$ 

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Edge operations.

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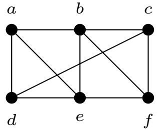
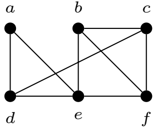
$G =$ 

then

$G - (a{-}b) =$ 

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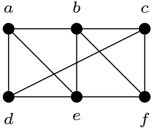
$G =$ 

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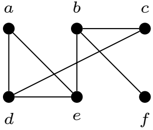
$G - \{a{-}b, c{-}f, e{-}f\} =$ 

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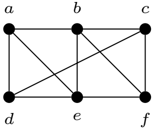
$G =$ 

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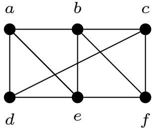
$G + (a{-}e) =$ 

Edge operations.

Let $\epsilon \in E$. There are two kinds of "contraction" of ϵ: contraction of graphs (allowing for multiple edges and loops) and contraction of simple graphs (not allowing for multiple edges and loops).

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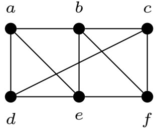
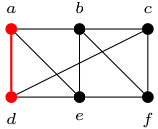
$G =$  and $\epsilon = a{-}d$, then

$G/\epsilon =$ 

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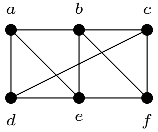
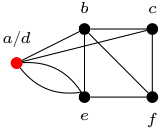
$G =$  and $\epsilon = a{-}d$, then

$G/\epsilon =$ 

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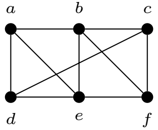
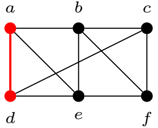
$G =$  and $\epsilon = a - d$, then

$(G/\epsilon)_{\text{simple}} =$ 

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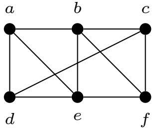

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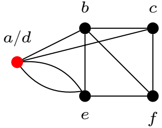

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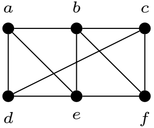
$G =$ 

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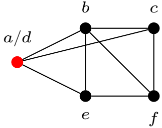
$(G/\epsilon)_{\text{simple}} =$ 

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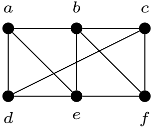
$G =$ 

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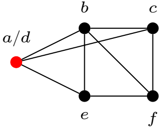
$(G/\epsilon)_{\text{simple}} =$ 

Note that G/e and $(G/\epsilon)_{\text{simple}}$ are not in general subgraphs of G.

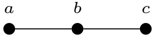
Unions

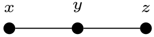
The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is

$$G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2).$$

Examples:

If

$G_1 =$ 

and $G_2 =$ 

Unions

The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is

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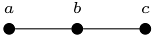
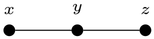
$G_1 =$ 

and $G_2 =$ 

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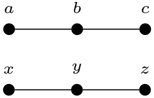
$G_1 \cup G_2 =$ 

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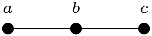
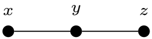
$G_1 =$ 

and $G_2 =$ 

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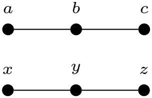
$G_1 \cup G_2 =$ 

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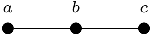
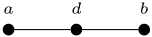
$G_1 =$ 

and $G_2 =$ 

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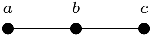
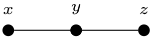
$G_1 =$ 

and $G_2 =$ 

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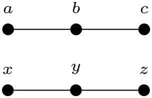
$G_1 \cup G_2 =$ 

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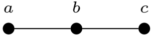
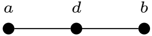
$G_1 =$ 

and $G_2 =$ 

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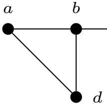
$G_1 \cup G_2 =$ 

Complements

Consider G as a subgraph of $K[V]$, the complete graph on the vertex set V. The complement of the graph G is

$$\bar{G} = (V, E_{K[V]} - E).$$

In other words, G and $\bar{G}$ have the same vertex set, but u and v are adjacent in $\bar{G}$ if and only if u and v are not adjacent in G.

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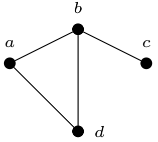
$G =$ 

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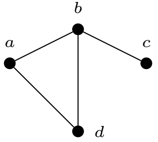
$G =$ 

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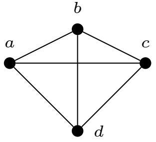
$K[V] =$ 

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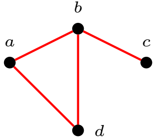
$G =$ 

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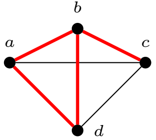
$K[V] =$ 

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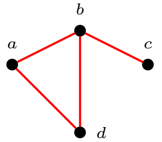
$G =$ 

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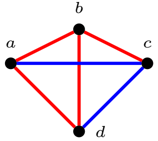
$K[V] =$ 

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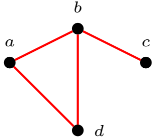
$G =$ 

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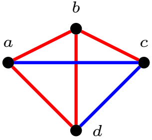
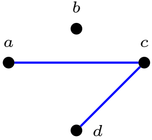
$K[V] =$ 

so $\overline{G} =$ 

Connectedness

Let $G = (V, E)$ be a graph. A walk is an alternating sequence of vertices and edges

$$w = (v_0, e_1, v_1, e_2, \cdots, e_n, v_n)$$

such that e_i has endpoints v_{i-1} and v_i. We say w has length n.

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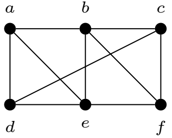
$G =$  ,

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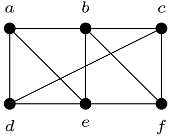

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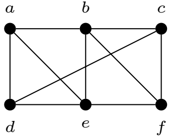
$G =$  ,

the walk $(a$

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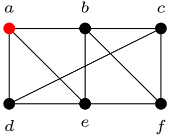

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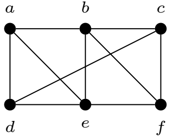
$G =$  ,

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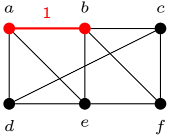

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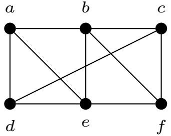
$G =$ 

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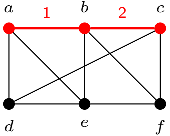

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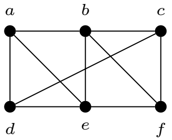
$G =$  ,

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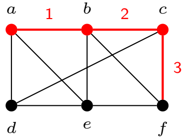

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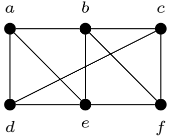
$G =$ ,

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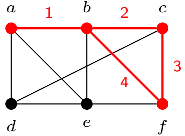

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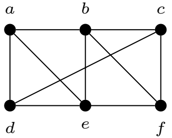
$G =$ 

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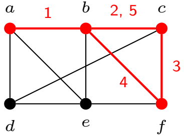

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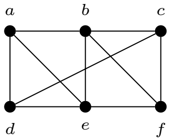
$G =$  ,

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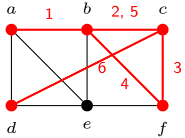

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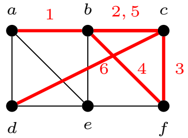
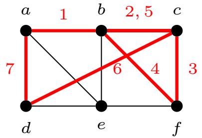
Walk:
a, b, c, f, b, c, d



Circuit:
a, b, c, f, b, c, d, a



Path:
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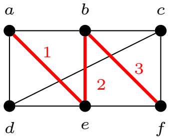

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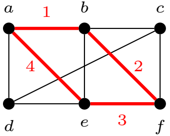
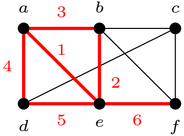
Cycle:

a, b, f, e, a



Trail:

a, e, b, a, d, e, f



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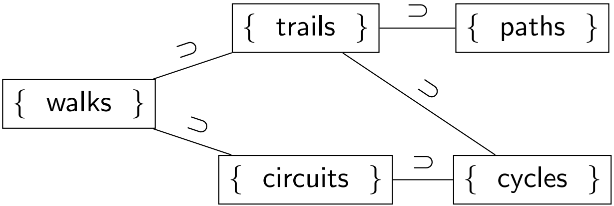

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1. A closed walk or circuit is a walk where $v_0 = v_n$.
2. A path is a walk so that no vertices (and therefore edges) are repeated.
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Note:

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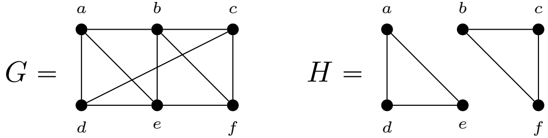

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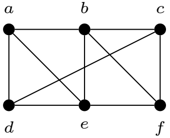

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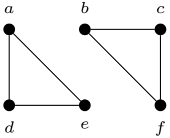

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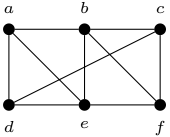

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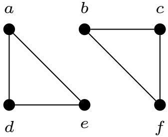

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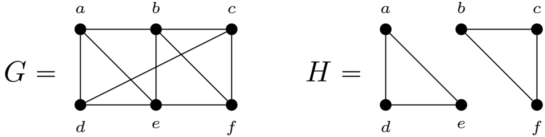

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A connected component of a graph is a maximally connected subgraph of G (H above has two connected components). Note that every connected component (or union of connected components) is an induced subgraph. Further, if W is the set of vertices in some connected component, then the induced subgraph H by W has the property that the degrees of the vertices in H are the same as the degrees of the corresponding vertices in G.

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