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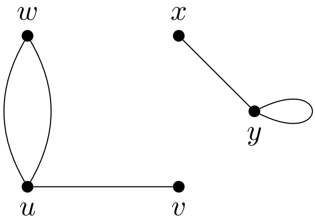

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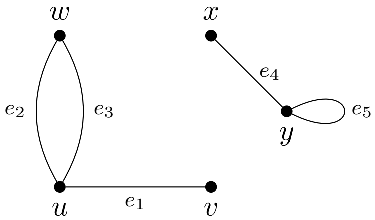

Here, the vertices are $V = \{u, v, w, x, y\}$, and the edges are
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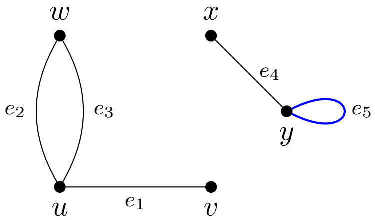

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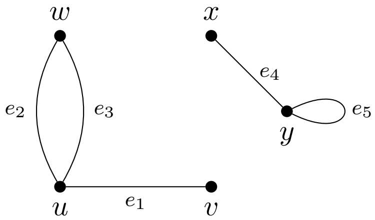
Example: 

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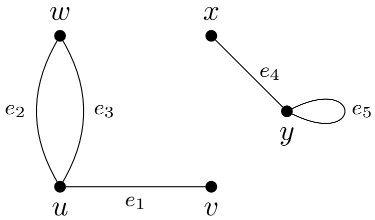

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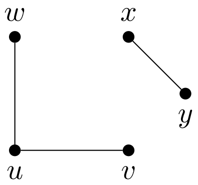

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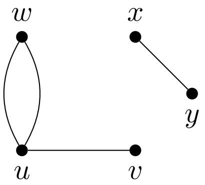

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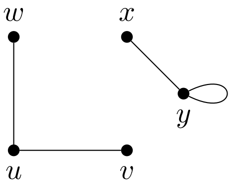

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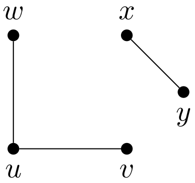

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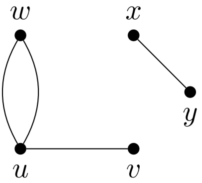

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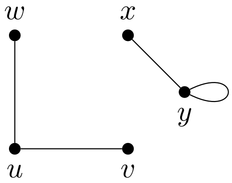

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So

$$\{ \text{ pseudographs/graphs } \} \supsetneq \{ \text{ multigraphs } \} \supsetneq \{ \text{ simple graphs } \}.$$

(Note: The $\supsetneq$ symbol is used here because, for example, every simple graph is a multigraph, but there are multigraphs that are not simple.)

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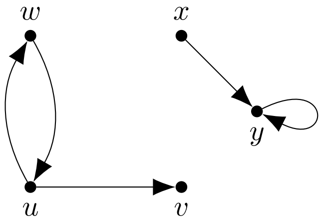

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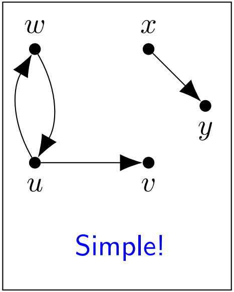

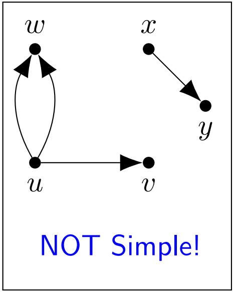

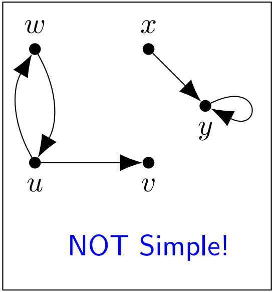

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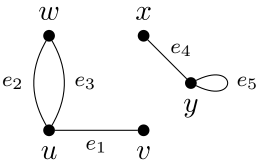
The book also talks about mixed graphs, where some of the edges are directed and some aren't. We usually take care of this by modeling the non-directed edges with *two directed edges*, one in each direction.

$G =$ 

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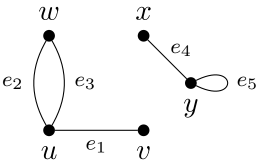
$G =$ 

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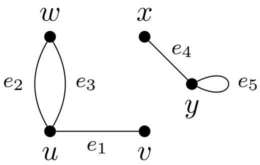
$G =$ 

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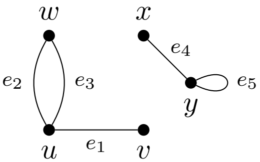
$G =$ 

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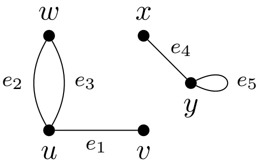

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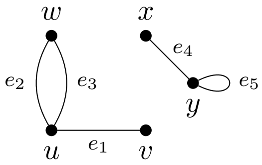
$G =$ 

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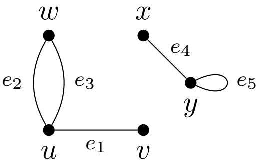

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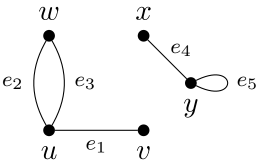
$G =$ 

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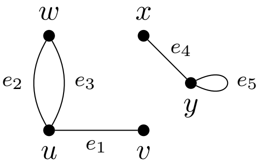
$G =$ 

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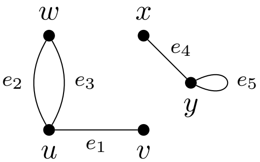
$G =$ 

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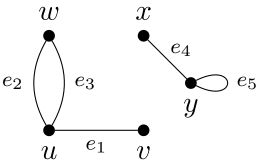
$G =$ 

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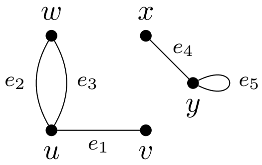
$G =$ 

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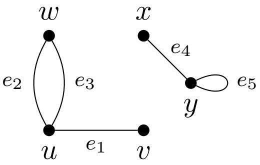

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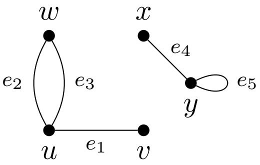
$G =$ 

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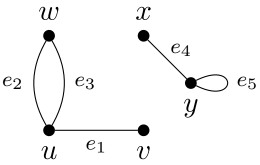
$G =$ 

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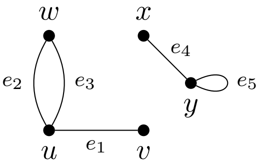
$G =$ 

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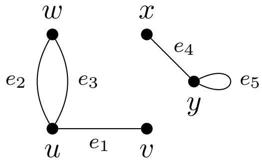

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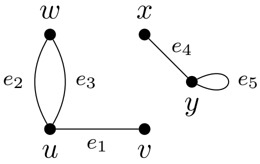
The degree $\deg(v)$ of a vertex v is the number of edge ends attached to v.

$G =$ 

$$
\begin{array}{ll}
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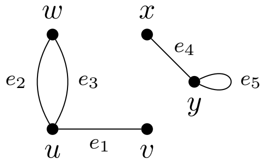
$G =$ 

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Fact: $\deg(v) \geqslant |N(v)|$; and a graph is simple if and only if

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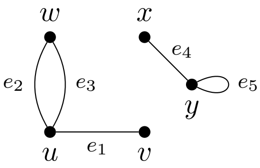

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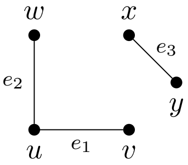

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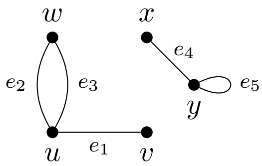
$G =$ 

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N(y) = \{x, y\} & \deg(y) = 3
\end{array}
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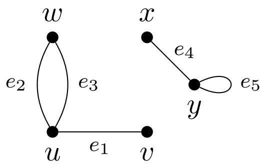
The degree $\deg(v)$ of a vertex v is the number of edge ends attached to v. We call a graph regular if all the vertices have the same degree.

$G =$ 

$N(u) = \{v, w\}$	$\deg(u) = 3$
$N(v) = \{u\}$	$\deg(v) = 1$
$N(w) = \{u\}$	$\deg(w) = 2$
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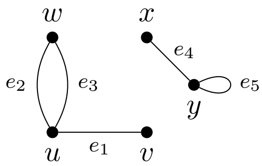
Theorem (The handshake theorem)

In a graph $G = (V, E)$*,*

$$2|E| = \sum_{v \in V} \deg v.$$

$G =$ 

$$\begin{array}{ll} N(u) = \{v, w\} & \deg(u) = 3 \\ N(v) = \{u\} & \deg(v) = 1 \\ N(w) = \{u\} & \deg(w) = 2 \\ N(x) = \{y\} & \deg(x) = 1 \\ N(y) = \{x, y\} & \deg(y) = 3 \end{array}$$

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Theorem (The handshake theorem)

In a graph $G = (V, E)$,

$$2|E| = \sum_{v \in V} \deg v.$$

Corollary

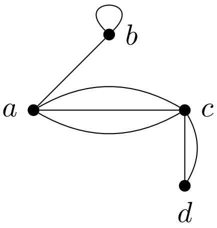
In any graph, there are an even number of odd vertices.

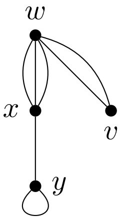
Graph isomorphisms

We say two graphs G and G' are isomorphic if there is a relabeling of the vertices of G that transforms it into G'. In other words, there is a bijection

$$f : V \to V'$$

such that the induced map on E is a bijection $f : E \to E'$.
For example,

$G =$ 

and $G' =$ 

are isomorphic via the map (doesn't depend on the drawing)

$$a \mapsto x, \quad b \mapsto y, \quad c \mapsto x, \quad d \mapsto v.$$

Recall: an equivalence relation on a set $\mathcal{A}$ is a pairing $\sim$ that is reflexive ($a \sim a$), symmetric ($a \sim b$ iff $b \sim a$), and transitive ($a \sim b$ and $b \sim c$ implies $a \sim c$).

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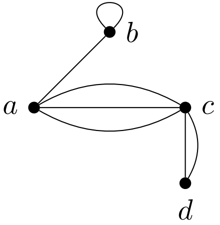
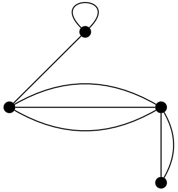
$G =$ 

is  .

Special graphs

Cycles. A cycle C_n is the equivalence class of simple graphs on n vertices $\{v_1, v_2, \ldots, v_n\}$ so that v_i is adjacent to $v_{i\pm1}$ (v_1 is adjacent to v_n).

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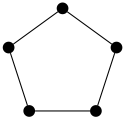

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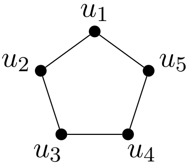

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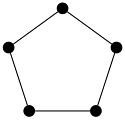

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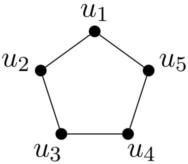

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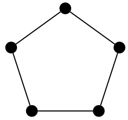

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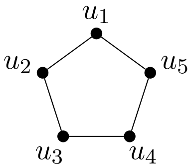

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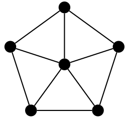

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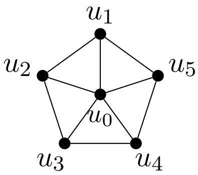

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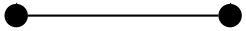
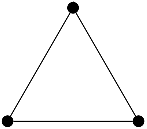
$K_2 =$ 

$K_3 =$ 

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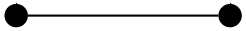
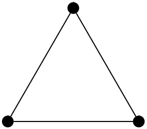
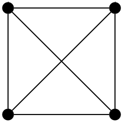
$K_2 =$ 

$K_3 =$ 

$K_4 =$ 

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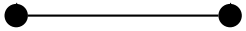
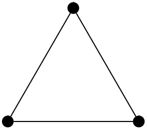
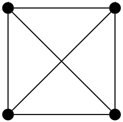
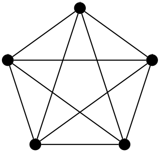
$K_2 =$ 

$K_3 =$ 

$K_4 =$ 

$K_5 =$ 

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In particular, for any $m \geqslant n \geqslant 1$, the complete bipartite graph $K_{n,m}$ is the class of simple graphs corresponding to the graph with vertices $V = V_1 \cup V_2$, where

$$V_1 = \{v_1, \ldots, v_n\} \quad V_2 = \{u_1, \cdots u_m\}$$

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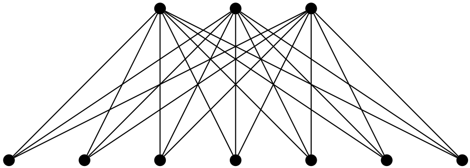
$K_{7,3} =$ 

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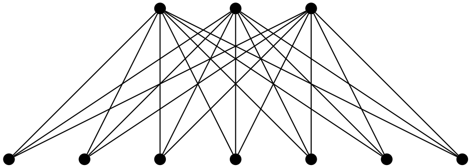
$K_{7,3} =$ 

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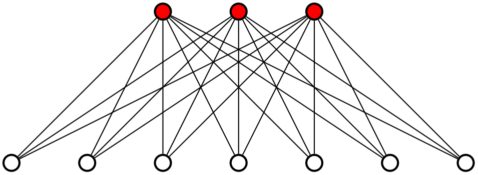

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Hypercubes. Let Q_n be the graph with vertex set

$$V = \{ \text{ bit strings (1's and 0's) of length } n \}$$

and edge set

$$E = \{u{-}v \mid u \text{ and } v \text{ differ in exactly one bit } \}.$$

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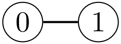
$Q_1 =$ 

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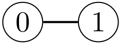
$Q_1 =$ 

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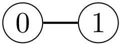
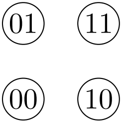
$Q_1 =$ 

$Q_2 =$ 

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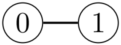
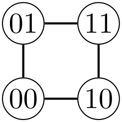
$Q_1 =$ 

$Q_2 =$ 

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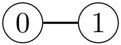
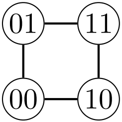
$Q_1 =$ 

$Q_2 =$ 

Q_3

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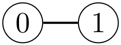
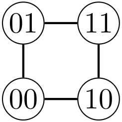
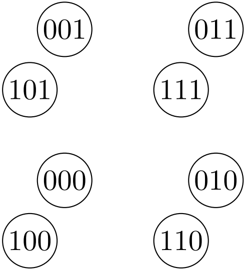
$Q_1 =$ 

$Q_2 =$ 

$Q_3 =$ 

Bipartite graphs. A graph is bipartite if V can be partitioned into two nonempty subsets V_1 and V_2 so that no vertex in V_i is adjacent to any other vertex in V_i for $i = 1$ or 2.

One way to show that a graph is bipartite is to "color" the vertices two different colors, so that no two vertices of the same color are adjacent.

Hypercubes. Let Q_n be the graph with vertex set

$$V = \{ \text{ bit strings (1's and 0's) of length } n \ \}$$

and edge set

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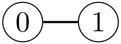
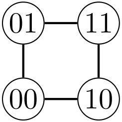
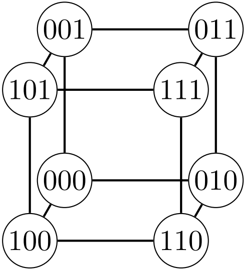
$Q_1 =$ 

$Q_2 =$ 

$Q_3 =$ 

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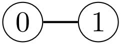
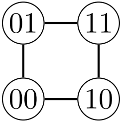
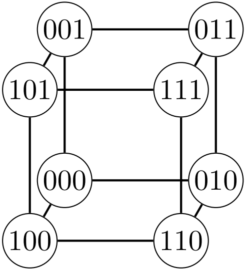
$Q_1 =$ 

$Q_2 =$ 

$Q_3 =$ 

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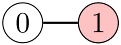
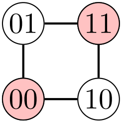
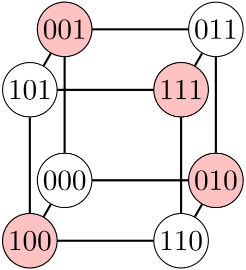
$Q_1 =$ 

$Q_2 =$ 

$Q_3 =$ 

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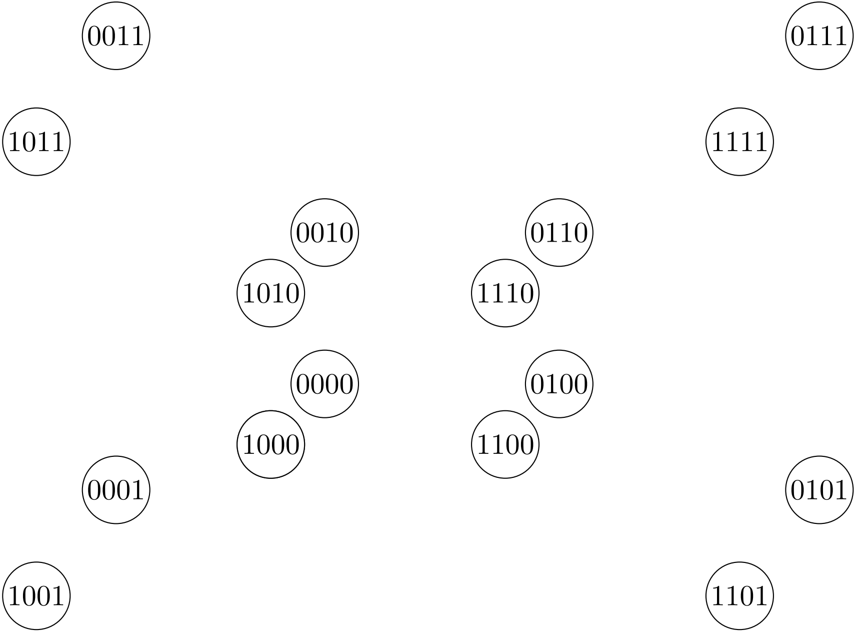
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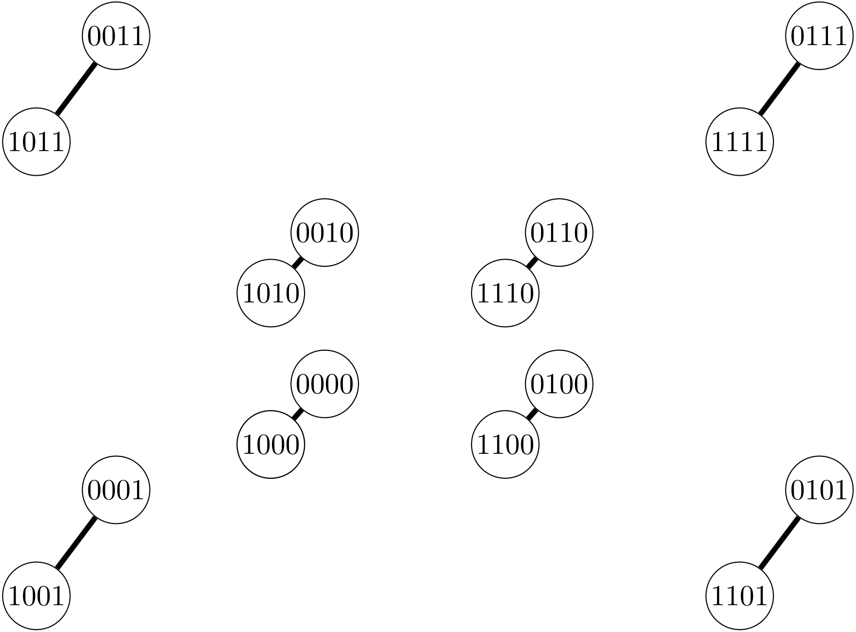
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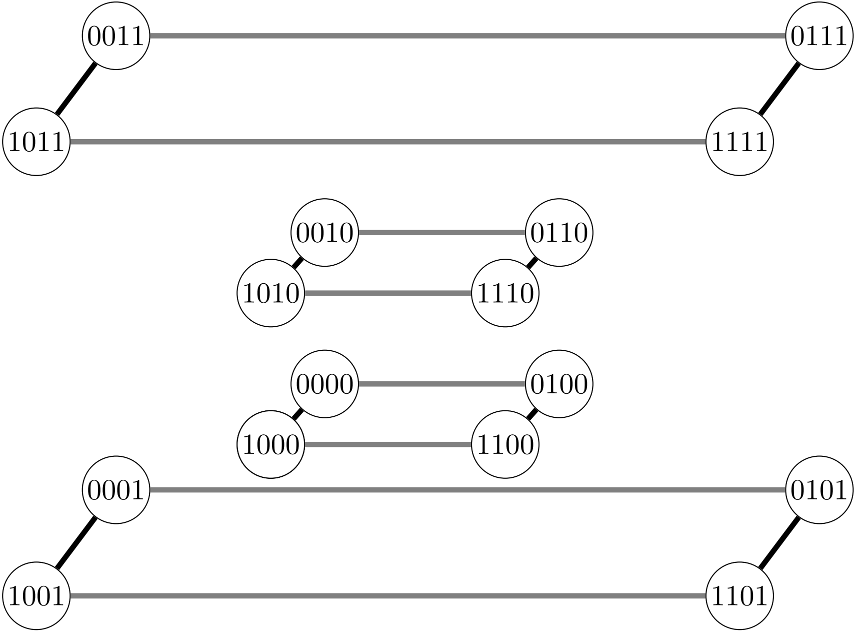
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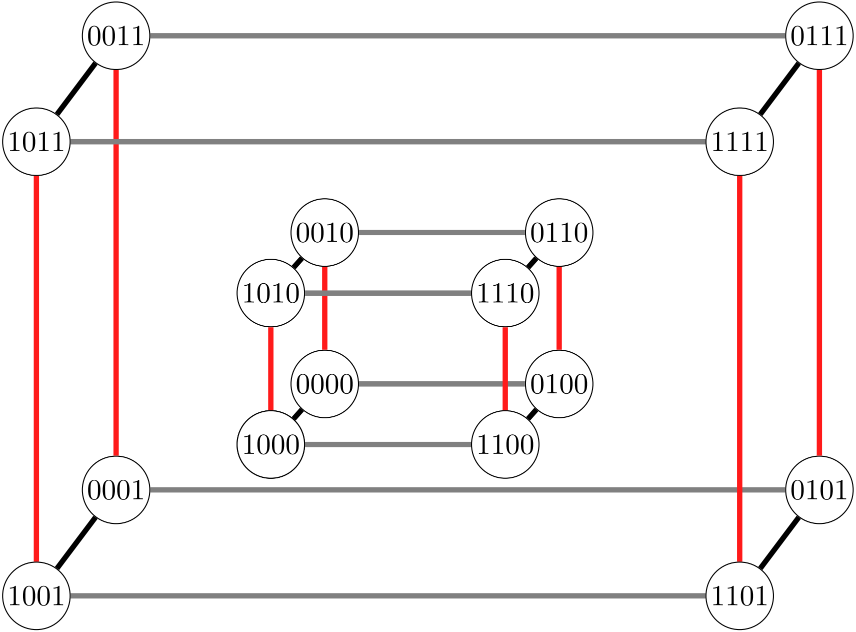
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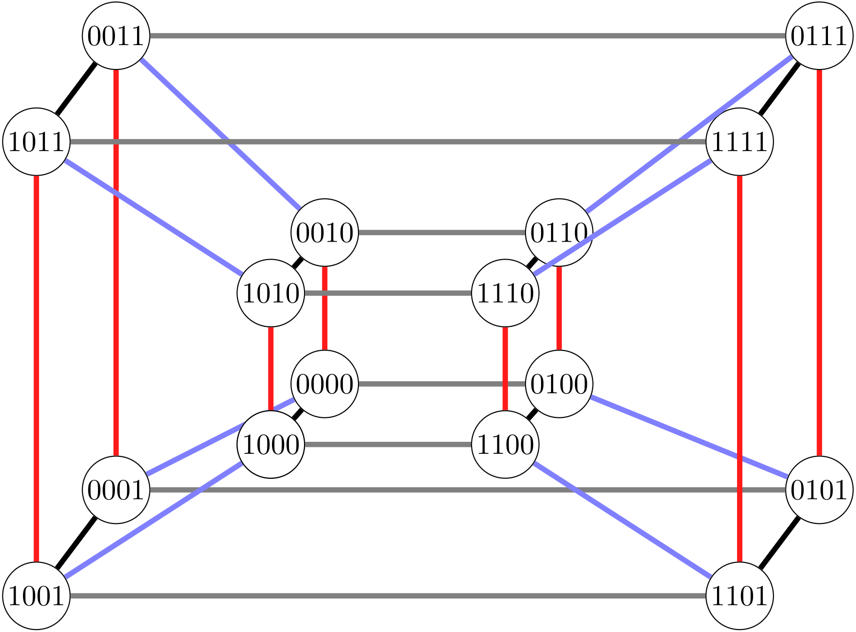
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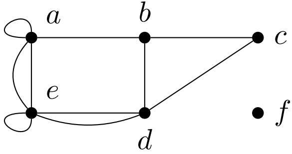
$G =$  is

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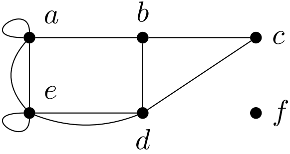
$G =$ 

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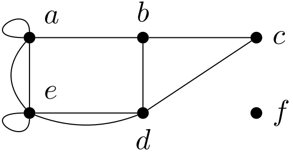
$G =$ 

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(Again, if the degree sequences of G and H differ, then $G \ncong H$. But if the degree sequences match, the *might* be isomorphic, but they *might not be*.)

Graph invariants

For example, consider the graphs

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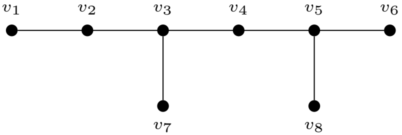

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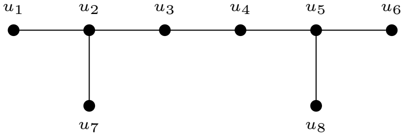

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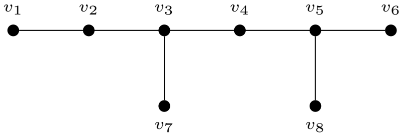
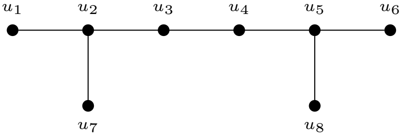
$G =$ 

$H =$ 

Both of these graphs have the degree sequence $3, 3, 2, 2, 1, 1, 1, 1$.

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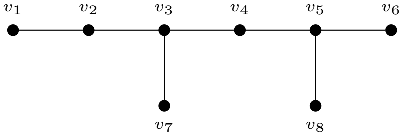
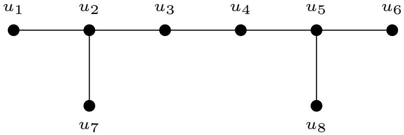
$G =$ 

$H =$ 

Both of these graphs have the degree sequence $3, 3, 2, 2, 1, 1, 1, 1$. But in G, there's a vertex of degree 1 adjacent to a vertex of degree 2, where as no vertex of degree 1 is adjacent to a vertex of degree 2 in H. So $G \ncong H$.