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The product rule: Suppose a procedure can be broken into a sequence of two tasks.

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Example: How many functions are there from a set of size three to a set of size 5?

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Sets: In set language, the product rule is the same as the rule for computing the size of a cartesian product of finite sets $A_1, \ldots, A_n$:

$$|A_1 \times A_2 \times \cdots \times A_n| = |A_1||A_2| \cdots |A_n|.$$

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The sum rule: If a task can be done either in one of n_1 ways or in one of n_2 ways, where there is no overlap in the n_1 and n_2 ways, then there are $n_1 + n_2$ ways to do the task.

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Example: If a student council member is going to be chosen from the first and second year student body, where there are 1503 first-year students and 1475 second-year students, how many possible candidates are there?

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Sets: In set language, the sum rule is the same as the fact that for some pairwise disjoint finite sets $A_1, \ldots, A_n$, i.e. $A_i \cap A_j = \varnothing$ for all $i \neq j$, we have

$$|A_1 \cup A_2 \cup \cdots \cup A_n| = |A_1| + |A_2| + \cdots + |A_n|.$$

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Say passwords for a site are required to be 6-8 characters long, using upper and/or lower case letters and/or numbers. How many possible passwords are there?

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So in total, there are $62^6 + 62^7 + 62^8$ possible passwords. (sum)

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You try: In-class exercise 19.

More rules

The subtraction rule: If a task can be done in either n_1 ways or n_2 ways, then the number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are common to the two different ways.

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$\boxed{120 + 200 - 15}$

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valid passwords in total (sub). //

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In set theory language, we call this the inclusion-exclusion principle:

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|.$$

Sometimes we say we "double counted", and have to fix it.

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Answer: Using the product rule, we can choose the cards in $52 * 51$ ways if we draw them one at a time in order $(n = 52 * 51)$.

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Answer: Using the product rule, we can choose the cards in $52 * 51$ ways if we draw them one at a time in order $(n = 52 * 51)$. But for each pair $\{\text{card}_A, \text{card}_B\}$, there are two ways to get that pair in this way:

card_A first, card_B second, or
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(So $d = 2.$)

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(So $d = 2$.) Answer: $\boxed{52 * 51/2}$

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Example: How many ways can you choose a committee of 3 from 10 people?

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Answer: Using the product rule, we can choose the people in $10 * 9 * 8$ ways if we choose them one at a time in order ($n = 10 * 9 * 8$).

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there are $3 * 2 * 1$ ways to get that committee in this way:

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then any 2 of the remaining second,
and whoever is left last.

(So $d = 3!$.) Answer: $\boxed{(10 * 9 * 8)/(3!)}$

More rules

The division rule: There are n/d ways to do a task if it can be done using a procedure that can be carried out in n ways, but for each way w, d of the ways have the same outcome w.

Example: How many ways can you choose a committee of 3 from 10 people?
Answer: Using the product rule, we can choose the people in $10 * 9 * 8$ ways if we choose them one at a time in order ($n = 10 * 9 * 8$). But for each committee

$$\{\text{member}_A, \text{member}_B, \text{member}_C\},$$

there are $3 * 2 * 1$ ways to get that committee in this way:

any 3 of them could have been chosen first,
then any 2 of the remaining second,
and whoever is left last.

(So $d = 3!$.) Answer: $\boxed{(10 * 9 * 8)/(3!)}$

You try: In-class exercise 20.