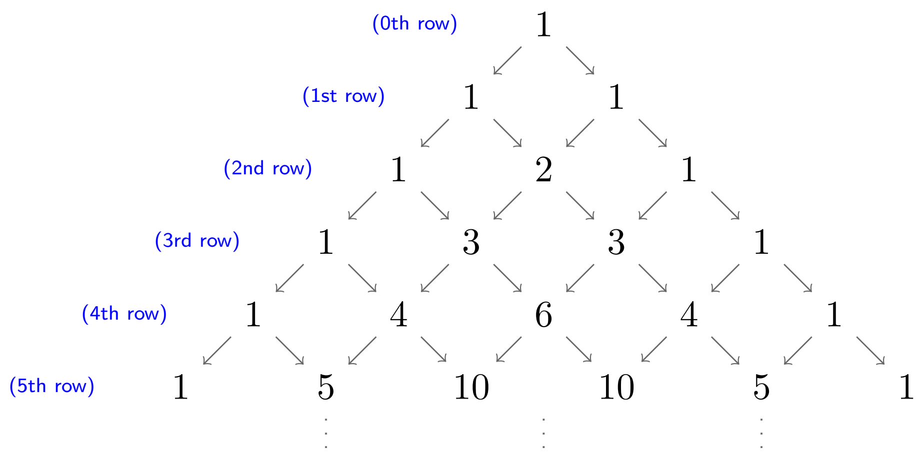
Welcome to Math 365!

Warm up:

1. How many ways can you choose 2 things from a set of 4? (Example, pick a committee of 2 people from a group of 4. This is different from the number of ways to choose a president and a vice president from a group of 4 people.)
2. How many ways can you choose 3 things from a set of 5?
3. Explain why there are exactly the same number of ways to choose 1 thing from a set of 5 as there are ways to choose 4 things from a set of 5.
4. How many ways are there to choose 3 things from a set of 3? 4 things from a set of 4? 5 things from a set of 5?
5. How many ways are there to choose 0 things from a set of 3? 0 things from a set of 4? 0 things from a set of 5?
6. Expand:
$$(1+x)^2 = 1 + 2x + x^2$$
$$(1+x)^3 =$$
$$(1+x)^4 =$$

Pascal's Triangle

Start and end each row with a 1. The ith row (starting with the 0th row) has $i+1$ entries. The middle entries are acquired by adding successive entries in the previous row.



Define:

ways to choose k things from n = "n choose k" $= \binom{n}{k}$.

Claim: $\binom{n}{k}$ is the kth entry of the nth row of Pascal's triangle

Course info

Me: Professor Daugherty, zdaugherty@gmail.com
Website:
https://zdaugherty.ccnysites.cuny.edu/teaching/m365s19/
Textbook: Discrete Mathematics and Its Applications (7th edition), by Kenneth Rosen.

Summary of syllabus (READ WEBSITE!!):
Grades: Homework&Quizzes: 20%, Exams: 25%/25%/30%.
Homework: due on Wednesdays in class, graded by completion. Posted on course website. FINAL DRAFTS.
Exams: Three exams, the last of which will be on the last day of class. You highest score will count for 30%.
Quizzes: In class, first on Wednesday 2/6.

Homework 0: Before class on Monday 2/4, send me an email at zdaugherty@gmail.com with subject line "Math 365: Homework 0", answering the questions outlined on the website.

Course expectations

- Read posted sections before class, and **bring your own copy of daily handouts and notes** (posted night before class).
- Come to class, participate, ask questions, work (possibly together) on in-class exercises.
- Come to office hours at least once in the semester (worth one homework assignment). If you can't make my office hour, make an appointment.
- Out of class studying and work should be 2–3 times the amount of time spent in class ($6.5 <$ hours/week). Find classmates to study and work with!
- Hand in "final draft" homework, on time. Get good practice with writing; using words and complete sentences—see Writing Exercise. Ok to work with other people, but write-ups should be your own. Homework submitted in LaTeXreceives 10% extra credit.
- If there are accessibility accommodations or exam conflicts to be organized, contact me as soon as possible.
- If you send me email, use complete sentences and be specific (ok to send pics of work!).

Definition

A set is an unordered collection of distinct objects.

(Contrast: a list is an ordered collection of objects)

Example

$$A = \{1, 2, 3\} = \{2, 1, 3\} = \{3, 2, 1\} \neq \{a, b, c\}.$$

Some special sets:

notation	definition	some terms
$\mathbb{Z}$	Integers	$0, \pm 1, \pm 2$
$\mathbb{Z}^+, \mathbb{Z}_{>0}$	Positive integers	$1, 2, 3$
$\mathbb{Z}_{\geqslant 0}$ ($\mathbb{N}$)	Natural numbers	$0, 1, 2, 3$
$\mathbb{Q}$	Rational numbers (fractions)	$0, 1, -1/2, 15/1004$
$\mathbb{R}$	Real numbers	$0, 1, 1/3, \pi, -\sqrt{2}$
$\mathbb{C}$	Complex numbers	$0, 1, -1/3, i = \sqrt{-1}, 5 + i\pi$
$\varnothing$	The empty set	(nothing is in here)

$$\text{Notice:} \quad \mathbb{C} \supsetneq \mathbb{R} \supsetneq \mathbb{Q} \supsetneq \mathbb{Z} \supsetneq \mathbb{Z}_{\geqslant 0} \supsetneq \varnothing.$$

(Notation: $\subseteq$ means subset, $\subsetneq$ means proper subset, and $\nsubseteq$ means not a subset. The symbol $\subset$ is unclear, and we try not to use it in this class.)

Notation:

$$\Big\{ \underbrace{\qquad\qquad}_{\text{objects}} \;\Big|\; \underbrace{\qquad\qquad}_{\text{conditions}} \Big\}.$$

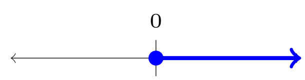
Read $|$ as "such that" or "that satisfy". Also useful:

$\in$ means "in" or "is an element of".

(Avoid using too much abbreviation in your writing though!)

Example: As a subset of $\mathbb{R}$, graph

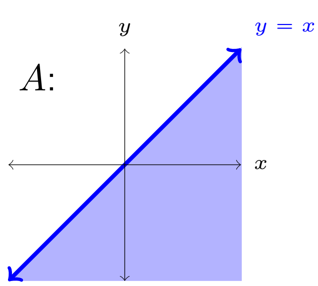
$\mathbb{R}_{\geqslant 0} = \{x \in \mathbb{R} \mid x \geqslant 0\}$

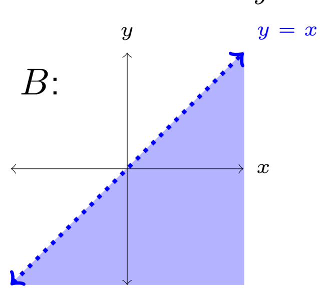


Example: As subsets of $\mathbb{R}^2$, graph

$$A = \{(x, y) \mid x, y \in \mathbb{R}, x \geqslant y\} = \mathbb{R}^2_{x \geqslant y}$$

$$B = \{(x, y) \mid x, y \in \mathbb{R}, x > y\} = \mathbb{R}^2_{x > y}$$

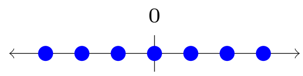




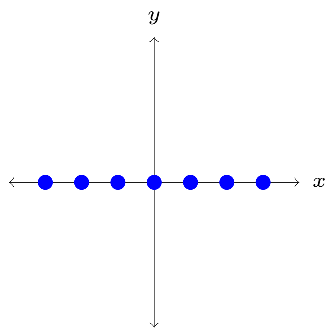
Context matters! To graph a set, we have to define both the set itself as well as the universal set it is contained in.

Example

As a subset of $\mathbb{R}$, $\mathbb{Z}$ looks like



As a subset of $\mathbb{R}^2$, $B = \{(x, 0) \mid x \in \mathbb{Z}\}$ looks like



Let A and B be sets.

(i) The (Cartesian) product of A and B, denoted $A \times B$, is
$$A \times B = \{(a, b) \mid a \in A, b \in B\}.$$

(ii) The power set of A, denoted $\mathcal{P}(A)$, is the set of subsets of A,
$$\mathcal{P}(A) = \{X \subseteq A\}$$
Note that $\varnothing$ and A are always elements of $\mathcal{P}(A)$.

(iii) The union of A and B is
$$A \cup B = \{c \mid \ c \text{ in } A \text{ or } B \text{ or both }\}.$$

(iv) The intersection of A and B is
$$A \cap B = \{c \mid \ c \text{ in } A \text{ and } c \text{ in } B \}.$$

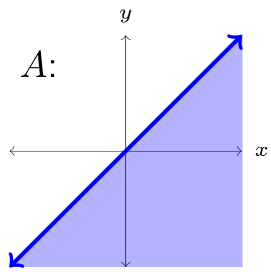
(v) The difference of A and B is
$$A - B = \{a \in A \mid a \text{ not in } B\}.$$

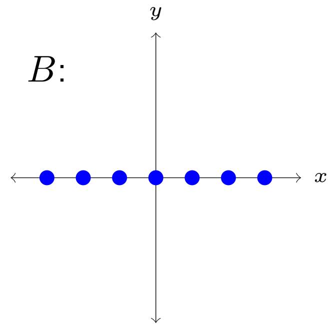
(vi) After choosing the universal set for A, the complement of A (in U) is
$$\overline{A} = U - A = \{u \in U \mid \ u \text{ is not in } A \}.$$

(vii) If A has a finite number of distinct elements, the cardinality of A, denoted $|A|$, is the number of those elements. Otherwise, we say A is infinite.

Example

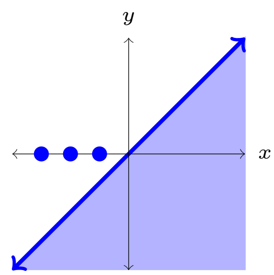
Let $A = \mathbb{R}^2_{x \geqslant y}$, $B = \mathbb{Z} \times \{0\}$, and $U = \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$.





Then we have

$$A \cup B = \{x \in U \mid x \in A \text{ or } B\} =$$



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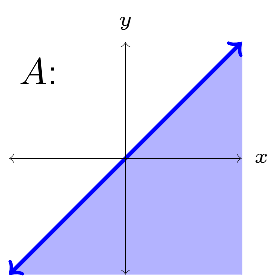

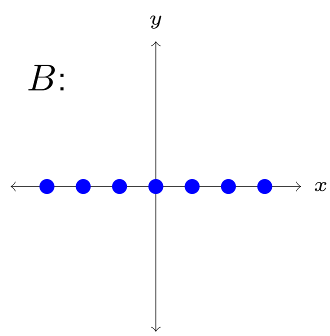

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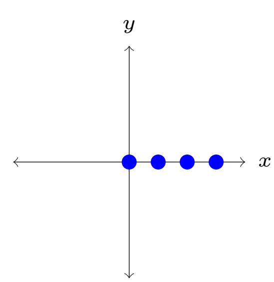
$$A \cap B = \{x \in U \mid x \text{ in both } A \text{ and } B\} =$$



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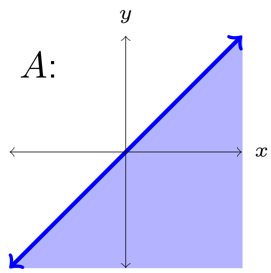
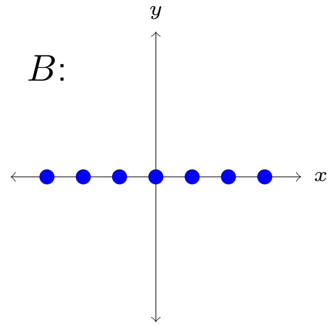
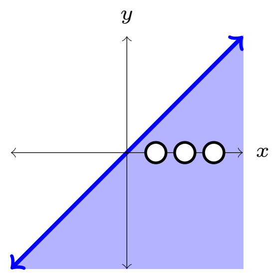




Then we have

$A - B = \{x \in A \mid x \notin B\} =$ 

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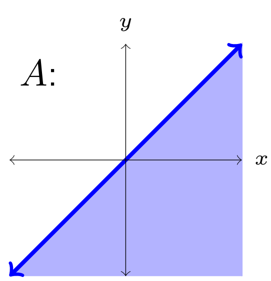
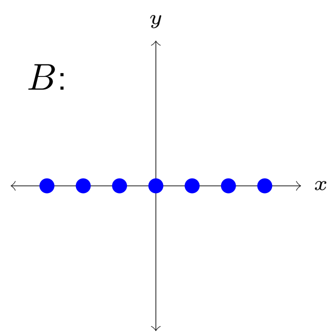
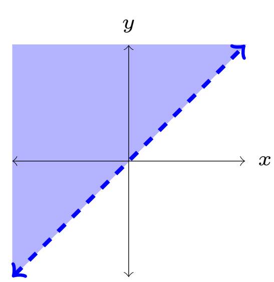




Then we have

$\overline{A} = U - A =$ 

Welcome to Math 365!

Course website (including syllabus): https://zdaugherty.ccnysites.cuny.edu/teaching/m365s19/
Professor: Zajj Daugherty, zdaugherty@gmail.com
Textbook: Discrete Mathematics and Its Applications (7th edition), by Kenneth Rosen.

Homework 0: due Monday 2/4 by email. (See course website.)

Attach at the end of Homework 1:

Before writing up your homework, read handouts "Communicating Mathematics through Homework and Exams" and "Some Guidelines for Good Mathematical Writing". Then, later, at the end of your write-up, include the following, labeling this as **"Writing exercise"**.

(a) List three things you learned or thought about more carefully after reading these documents.

(b) Mark up this written homework assignment, showing where you followed or failed to follow the mechanical and stylistic issues outlined in *Communicating Mathematics. . . .* (This means treat your write-up as a rough draft, and go point-by-point through this handout and address instances where on your assignment you followed or did not follow each direction. Use a different-colored pen if you have one.)
How might you improve in the future?

(c) List three or more ways that you succeeded or failed at following the advice in *Some Guidelines. . . .* How might you improve in the future?

To receive any credit for homework 1, you must do this writing exercise.

Exercise 1.

(a) Use set-builder notation, i.e. { elements | conditions }, to write the following sets.
  (i) The set of positive integers that are multiples of 5.
  (ii) The set of real numbers that are not integers.
  (iii) The set of rational number that are between -3 and 19, inclusive.

(b) Let $U = \{-3, -2, -1, 0, 1, 2, 3\}$ and $V = U \times U$. List the elements in the following sets
  (i) $A = \{x \in \mathbb{Z} \mid -2 \leq x < 2\}$
  (ii) $B = \{x \in \mathbb{Z} \mid 0 < x < 3\}$
  (iii) $\mathcal{P}(B)$
  (iv) $A \cup B$
  (v) $A \cap B$
  (vi) $A - B$
  (vii) $\overline{A}$, where U is the universal set.
  (viii) $C = \{(x, y) \in A \times B \mid x \neq y\}$.
  (ix) $\overline{C}$, where V is the universal set.
  (x) $A \times A$
  (xi) $(A \times A) \cap C$
  (xii) $(A \times A) \cup C$
  (xiii) $\overline{A \times A} \cap C$, where V is the universal set.

(c) Pick a set S and two universal sets U_1 and U_2 that illustrate that $\bar{A}$ depends on the choice of universal set.

(d) Let A and B be sets contained in a universal set U. Decide whether the following identities are **true or false**. If false, give an example where the identity doesn't hold. If true, explain why (in complete sentences).
  (i) $A \cap B = B \cap A$
  (ii) $A \cup B = B \cup A$
  (iii) $A - B = B - A$
  (iv) $A \times B = B \times A$
  (v) $|A - B| = |A| - |B|$
  (vi) If A is finite, then so is $\mathcal{P}(A)$
  (vii) $\overline{A} \cap \overline{B} = \overline{(A \cup B)}$

Exercise 2. (a) Let X be the set of students who live within one mile of school and let Y be the set of students who walk home after school. Describe the students in each of these sets.

(i) $X \cap Y$
(ii) $X \cup Y$
(iii) $X - Y$
(iv) $Y - X$

(b) How many elements does each of these sets have (where a and b are distinct elements)?

(i) $P(\{a, b, \{a, b\}\})$
(ii) $P(\{\emptyset, a, \{a\}, \{\{a\}\}\})$
(iii) $P(P(\emptyset))$

(c) Suppose that $A \times B = \emptyset$, where A and B are sets. What can you conclude?

(d) Look up Pascal's triangle online. Give three interesting facts about it (not including any covered in class).

Due Wednesday 2/6: Handout Exercises 1–7; and the Writing Exercise.
For next time: read sections 2.1 and 2.2.

Some shorthands you'll see in the book:

$\in$ means "in", "contained in"	Ex: $x \in \mathbb{R}$ means x is a real number.
$\forall$ means "for all"	Ex: $A \subseteq B$ if $\forall a \in A$, we have $a \in B$.
$\wedge$ means "and" (both)	Ex: $A \cap B = \{x \in U \mid (x \in A) \wedge (x \in B)\}$.
$\vee$ means "or" (one or the other or both)	Ex: $A \cup B = \{x \in U \mid (x \in A) \vee (x \in B)\}$.
$\neg$ means "not"	Ex: $\overline{A} = \{x \in U \mid \neg(x \in A)\}$.