Computing data with spreadsheets

Example: Computing triangular numbers and their square roots.
Recall, we showed $1 + 2 + \cdots + n = \frac{n(n + 1)}{2}$.
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- C2: \texttt{=SQRT(B2)}
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The result should look like

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( n )</td>
<td>triangle</td>
<td>sqrt</td>
</tr>
<tr>
<td>2</td>
<td>( 1 )</td>
<td>( 1 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>3</td>
<td>( 2 )</td>
<td>( 3 )</td>
<td>1.73205081</td>
</tr>
<tr>
<td>4</td>
<td></td>
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<tr>
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<td>( \frac{A2 \times A2 + 1}{2} )</td>
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<td>( =A2+1 )</td>
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If you don’t want it to adjust part of a formula with shifts, add a $ in front of the row number, cell number, or both: A$2, $A2, or $A$2.
Pythagorean Triples

The Pythagorean theorem says the lengths of the sides of a right triangle satisfy the following:

\[ a^2 + b^2 = c^2 \]
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Geometric proof: Compare the area of the white spaces in

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Number theorists ask: Are there integer solutions?

Yes!

1. 3
2. 4
3. 5
4. 5
5. 12
6. 13
7. 8
8. 15
9. 17

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Trivial solution: \( a = b = c = 0 \).

(Don't forget to look for the simplest solutions!!)
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Ex: \((3,4,5)\)     Non-ex: \((6,8,10)\).
Primitive Pythagorean triples (PPTs)

More examples:

(3, 4, 5)  (20, 21, 29)  (28, 45, 53)  (5, 12, 13)
(9, 40, 41)  (33, 56, 65)  (8, 15, 17)  (7, 24, 25)
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\[
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$$(2k + 1)^2 = 4k^2 + 4k + 1 = 2 \underbrace{(2k^2 + 2k)}_{\text{integer}} + 1$$
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Similarly, you can show that sum of evens and odds follows the pattern:

\[
\begin{array}{c|cc}
+ & \text{even} & \text{odd} \\
\hline
\text{even} & \text{even} & \text{odd} \\
\text{odd} & \text{odd} & \text{even}
\end{array}
\]
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You try: For solutions $a, b, c \in \mathbb{Z}$ to $a^2 + b^2 = c^2$, complete the following table.

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Which could possibly be primitive?
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Without loss of generality (WLOG), assume $a$ is odd and $b$ is even. (WLOG here means that since $a$ and $b$ are interchangeable, we don’t need to also consider the cases where the reverse is true)
Looking for more primitive Pythagorean triples (PPTs)

Without loss of generality (WLOG), assume \(a\) is odd and \(b\) is even, so \(c\) is odd. (WLOG here means that since \(a\) and \(b\) are interchangeable, we don't need to also consider the cases where the reverse is true)

If \(a^2 + b^2 = c^2\), then \(a^2 = c^2 - b^2\)
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If \(a^2 + b^2 = c^2\), then \(a^2 = c^2 - b^2 = (c - b)(c + b)\).

Claims: Assume \(a, b, c > 0\).

1. Both \(c - b\) and \(c + b\) are positive odd integers.
Looking for more primitive Pythagorean triples (PPTs)

Without loss of generality (WLOG), assume $a$ is odd and $b$ is even, so $c$ is odd. (WLOG here means that since $a$ and $b$ are interchangeable, we don't need to also consider the cases where the reverse is true)

If $a^2 + b^2 = c^2$, then $a^2 = c^2 - b^2 = (c - b)(c + b)$.

Claims: Assume $a, b, c > 0$.

1. Both $c - b$ and $c + b$ are positive odd integers.
2. There are no divisors common to $c - b$ and $c + b$. 
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2. There are no divisors common to $c - b$ and $c + b$.
3. Both $c - b$ and $c + b$ are perfect squares.

Later we will talk about the uniqueness of prime factorizations. For today, assume that if

$$n = ab$$

for positive integers $n, a, \text{ and } b$,

then there is a one-to-one correspondence between

$$\{\text{prime divisors of } n, \text{ counting multiplicity}\} \quad \text{and} \quad \{\text{prime divs of } a, \text{ counting mult}\} \bigcap \{\text{prime divs of } b, \text{ counting mult}\}.$$
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Let \((a, b, c)\) be a PPT with \(a, b, c > 0\). We’ve shown that \(c - b\) and \(c + b\) are positive perfect squares with no common divisors.
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Define \(s, t \in \mathbb{Z}_{>0}\) by

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c + b = s^2 \quad \text{and} \quad c - b = t^2.
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**Theorem**

*Primitive Pythagorean triples are classified by positive integers \(a, b, c\) such that*

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a = st, \quad b = \frac{s^2 - t^2}{2}, \quad \text{and} \quad c = \frac{s^2 + t^2}{2},
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*for odd integers \(s > t \geq 1\) with no common factors.*
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You try: Exercise 5, parts (c), (d), and/or (e).