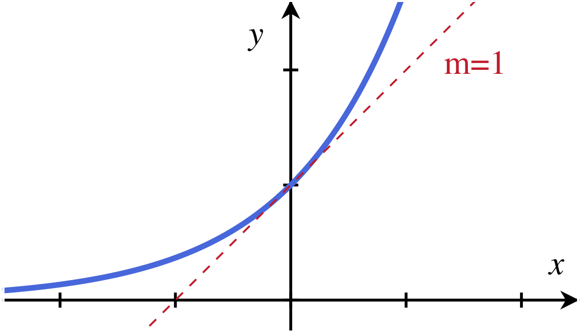
Derivatives of Exponential and Logarithm Functions

10/17/2011

The Derivative of $y = e^x$

Recall!

e^x is the unique exponential function whose slope at $x = 0$ is 1:



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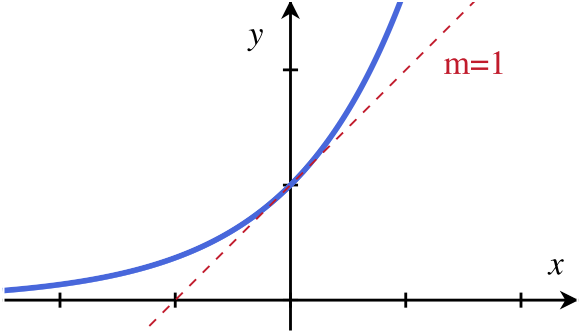

$$\lim_{h \to 0} \frac{e^{0+h} - e^0}{h} = \boxed{\lim_{h \to 0} \frac{e^h - 1}{h} = 1}$$

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So

$$\boxed{\frac{d}{dx} e^x = e^x}$$

The Chain Rule

Theorem

Let u be a function of x. Then

$$\frac{d}{dx}e^u = e^u\frac{du}{dx}.$$

Examples

Calculate...

1. $\frac{d}{dx}e^{17x}$

2. $\frac{d}{dx}e^{\sin x}$

3. $\frac{d}{dx}e^{\sqrt{x^2+x}}$

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Notice, every time:

$$\frac{d}{dx} e^{f(x)} = f'(x) e^{f(x)}$$

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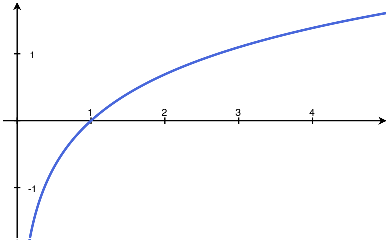
$$\boxed{\frac{d}{dx} \ln(x) = \frac{1}{x}}$$

Does it make sense?

$$\boxed{\frac{d}{dx}\ln(x) = \frac{1}{x}}$$

$f(x) = \ln(x)$



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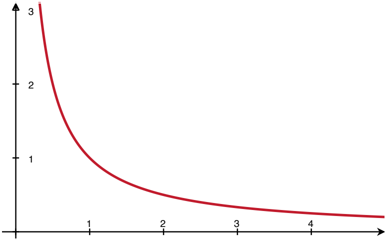

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Notice, every time:

$$\frac{d}{dx} \ln(f(x)) = \frac{f'(x)}{f(x)}$$

The Calculus Standards: e^x and $\ln x$

To get the other derivatives:

$$a^x = e^{x \ln a}$$

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This equation, $\frac{d}{dx}y = ky$ is an example of a *differential equation*.